Research on the Application of Fractional Calculus in Statistics

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Abstract: Fractional calculus has been widely used in various fields in China. In recent years, with the continuous maturity and development of fractional calculus theory and application, its application in statistics has an important role in texture. The theoretical and numerical algorithms of fractional calculus and differential equations are used in the field of statistical systems, which solves many problems in statistical systems. In this paper, based on the theory of the definition of fractional calculus, the fractional calculus the special function is expounded. At the same time, the application of fractional calculus in complex systems is specifically analyzed, and the application of non-Newton fluid mechanics and anomalous diffusion and random walk theory. With the deepening of the research on fractional calculus in academic circles and the expansion of its application scope, the role of fractional calculus in the statistical system has been further expanded, and will play a greater role in reference.

1. Introduction

Fractional calculus is closely related to Fresnel diffraction, Wigner distribution, wavelet transform, etc., and has been widely used in many fields of scientific research and engineering technology, for example, neural networks, differential equations, quantum mechanics, diffraction theory, optical systems. , optical imaging and radar, communications, sonar and other fields, become an interdisciplinary research direction. Due to the insubstantial distinction between microscopic and macroscopic time scales, many statistical systems need to be described by fractional calculus equations. For example, charge transport in amorphous semiconductors, diffusion of bottom water contaminants, relaxation of polymer systems, and trace dynamics of polymer networks. The common features of these systems are: slow diffusion and normal diffusion of Einstein, known as under-diffusion. The establishment of fractional diffusion equations by considering memory effects. They can be used to describe the anomalous diffusion of complex systems, such as the extension of polymers in the case of external potential fields and the trapping of carriers in deep wells in amorphous semiconductors. Through the deepening of research, fractional calculus is widely used in more fields.

2. Definition of fractional calculus

Fractional calculus is relative to traditional integer calculus, which is expressed in fractional calculus or fractional integral of a function, such as one-half integral, two-thirds differential, and so on. Western scientists have defined fractional calculus at the end of the 17th century, but the theory of integer calculus is in the initial stage of development.

According to the development of traditional calculus, fractional calculus has different definitions in different development periods. Compared with integer-order calculus, fractional-order calculus is an extension of the order of integral and differential. Scholars have a narrow and broad definition of the concept of fractional differential. In terms of narrow definitions, the fractional order of differential and integral in fractional calculus exponential calculations, for example, the application of fractional calculus is more often used in the second order differential or integral, that is, with traditional integers. Any fractional calculus in terms of calculus. In a broad sense, fractional calculus is not only a simple non-integer differential and integral for the “fractional” order, but a non-integer arbitrary calculus in the field of nonlinear mathematics, and conceptually it is simply a

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fractional order. Differential or integral calculus, but an arbitrary order calculus containing rational numbers and irrational numbers or even complex numbers. From another perspective, this is the derivative and development of the concepts and theories of fractional calculus. The concept of fractional calculus in the narrow sense has been defined by scholars in the field of mathematics in the early period. After years of development, the improvement of the social level has also enabled the concept to be supplemented and improved. However, due to historical reasons, people still use fractional calculus. A name is only broader in terms of concept definition. In addition, it should be understood that the differential operator in fractional calculus is a linear operator in the mathematical world.

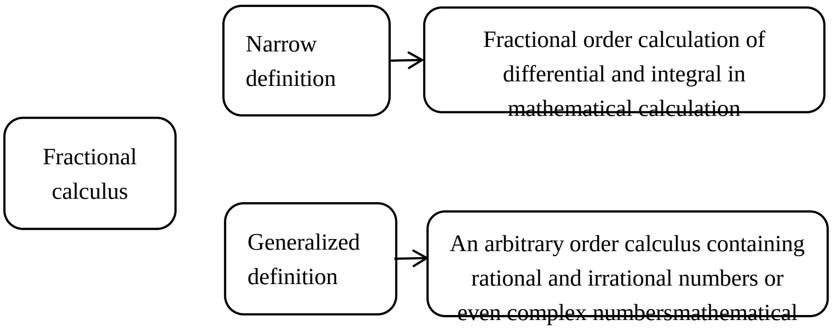

Figure 1 Fractional calculus definition

3. Special function

When solving micro-equations containing fractional-order operators, some integral transformations, such as Fourier transform, Laplace transform, Mellin transform, etc., are often used. When inverting these transforms, the solutions of fractional-order micro-equations are usually obtained. Contains special functions such as Mittag-Leffler functions and Fox H functions.

3.1 Mittag-Leffler function

The extension of the classical exponential function of the Mittag-Leffler function is defined as follows:

Among them α >0, and $z \in C$.

The two-parameter Mittag-Leffler function is defined as follows:

Among them α >0, and β , $z \in C$.

The three-parameter Mittag-Leffler function is defined as follows:

$$E_{\alpha,\ \beta}^{\gamma}(z) = \sum_{n=0}^{\infty} \frac{(\gamma)_n}{\Gamma(\alpha n + \beta) n!} \quad (\alpha,\ \beta,\ \gamma \in C, \mathrm{Re}(\alpha) > 0),$$

Among them, $(\gamma)_0 = 1$, and

the three-parameter Mittag-Leffler function can also be expressed as follows:

$$E_{\alpha,\ \beta}^{\gamma}(z) = \frac{1}{\Gamma(\beta)} + \sum_{k=1}^{\infty} \frac{\Gamma(\gamma + k) z^k}{\Gamma(\gamma)\Gamma(\alpha k + \beta)\Gamma(k+1)}, (\alpha, \beta, \gamma \in C, \mathrm{Re}(\alpha) > 0)$$

The four-parameter Mittag-Leffler function is defined as follows:

$$(\gamma)_{qn} = \frac{\Gamma(\gamma + qn)}{\Gamma(\gamma)}$$

Among them, $\mathrm{Re}(\alpha), \mathrm{Re}(\beta), \mathrm{Re}(\gamma) > 0, q \in (0,1) \cup N$, and is the generalized Pochhammer symbol.

3.2 Fox H-function

The Fox H-function plays a very important role in solving the solutions of fractional-order current differential equations in control and statistics. This function contains almost all special functions that are widely used in applied mathematics and statistics: Mittag-Leffler type functions, generalized transcendental geometry functions, generalized Bessel functions, and Meijer's G functions. The integral of the Fox H-function in the sense of Mellin-Barnes is expressed as:

Among them, $i=\sqrt{-1}$, $z \neq 0$, and

$z^s = \exp[sLog|z| + i\arg(z)]$, $Log|z|$ means natural logarithm of $|z|$, and $\arg(z)$ can not be the main value. In addition, Among them, m, n, p, q are non-negative integers, so $0 \le n \le p, 1 \le m \le q; A_j(j=1,2...,p), B_j(j=1,2,...,q)$ are positive numbers, $a_j(j=1,2,...,p)$, $b_j(j=1,2,...,q)$ is a set of plural. when $\gamma, \lambda = 0,1,2,...;$ $h=1,2,...,m;$ $j=1,2,...n$, $A_j(b_h+\gamma) \neq B_h(a_j-\lambda-1)$

$$s=\left(\frac{b_j+\gamma}{B_j}\right),(j=1,2,...,m;\gamma=0,1,...)$$

Last, L is a separate

$(\Gamma(b_j - B_j s)(j=1,2,...,m))$ and the pole of The line of the two sets of points

4. Application of fractional calculus in complex systems

Complex systems are a research area that scholars are very concerned about, and it is difficult to give precise definitions at present. But in general, it contains two aspects of ideological connotation. First, self-organized critical. Scholars believe that large dynamic systems always tend to a critical state without feature space time scale. Second, the principle of active walk, mainly refers to complex systems. Each unit in the medium exchanges information with its environment and with each other. The principle of active walk has been successfully applied with the formation of dielectric breakdown patterns, the transport of ions in the glass state, and the cooperation of ants in collecting food. Some scholars have defined the complex system as follows: it has long-term memory or long A range of spatially related systems.

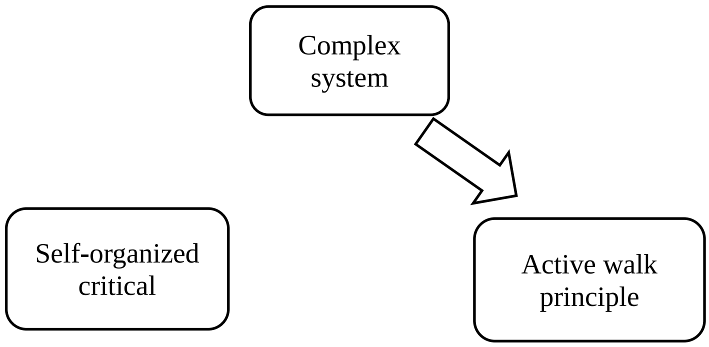

Figure 2 Definition of a complex system

4.1 Application of Fractional Calculus in Non-Newton Fluid Mechanics

Since viscoelastic materials can be divided into elastic solids and fluids, the application of fractional calculus is extended to the constitutive equation of one-dimensional scalar form of viscoelastic fluid, and the first-order derivative of integer order time is replaced by R-LFC, in some special Under the geometric edge condition, the fixed Cauchy problem can be obtained, and some special functions related to FC, such as H-Fox function, generalized Mittag-Leffler function and Wright function, can often get the analytical solution of the problem, and then reveal the stickiness. The elastic flow characteristics, and the fractional derivative, the resulting solution tends to the solution of the integer order Newton fluid. Chinese scholars have done a lot of research work in this area, and have achieved certain research results. The above analytical solution, the establishment of the steady state process satisfies the scale law, so that the main features of FC are maintained.

4.2 Application of Fractional Calculus in Anomalous Diffusion and Random Walk Theory

Applying fractional calculus to extend the classical integer order diffusion and wave partial differential equations to the fractional order of time and space, then extend to various nonlinear equations and give the solution of the initial boundary value problem. An important area of application of calculus applications, these issues have important logistics backgrounds, such as dispersion in fractal and porous media, semiconductor physics, turbulence and condensed matter, etc. Historically, the diffusion equation has been from two different perspectives. Established and developed. The first is that the first and second laws of Fick establish the constitutive relation between flux and flow to study the diffusion equation, which can be a deterministic view. The second is random walk. The early Einstein-Kolmogorov diffusion equation is a typical example. After establishing the generalized concepts of fractional constitutive relations and fractional random walks, the consistent forms of fractional diffusion equations are given simultaneously from these two directions. In general, the average squared displacement of time $< r^2(t) > \alpha t^{\lambda}$ Scale to characterize a fractional diffusion property. If $\lambda = 1$,diffusion of integer order Gauss; if λ < 1, λ >1representing abnormal sub-diffusion and abnormal hyper-diffusion. The concentration probability density distribution formed by the anomalous point source anomalous diffusion in the disordered fractal medium and the European space and gives the analytical expression of the scattering function. When the diffusion coefficient is a function of γ radius, Or the power function of the concentration C, although it is a fractional-order nonlinear equation, the application of the transform group technique can sometimes find a analytic solution with a material significance. At this time, most of the solutions have the characteristics of a class-like traveling wave, which is similar to the classic The diffusion equation is quite different, the former is limited in speed, and the latter is instantaneously transmitted to infinity.

5. Conclusion

With the development of society, the application of fractional calculus is more and more extensive, and the principle of fractional calculus is effectively applied in all fields of society. Especially for statistical systems, the complexity of fractional calculus operation is better to help solve it. The problem of the imaginary degree provides more practical possibilities for the theoretical study of fractional calculus. With the gradual expansion of fractional calculus theory and applied research, the performance of fractional calculus is maximized, and the statistical progress is accelerated. The fractional crisis segment will play a greater role in statistical system research.

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